MATHEMATICS

RESOURCE PACK

GRADE 11 TERM 1

EXPONENTS AND SURDS

RESOURCE 1

TOPIC 1: LESSON 1

	Law	Example	Explanation
1	$x^a \times x^b = x^{a+b}$	$2^3 \times 2^2 \times 2$ $= 2^{3+2+1}$ $= 2^6$	When multiplying powers with like bases keep the bases the same and add the exponents.
2	$\frac{x^a}{x^b} = x^{a-b}$	$\frac{6x^6}{2x^2} = 3x^4$	When dividing powers with like bases keep the base and subtract the exponent. Divide coefficients (numbers) as per normal.
3	$(x^a)^b = x^{ab}$	$(-2a^2b^3)^4$ $= (-2)^2 \times a^{2\times2} \times b^{3\times2}$ $= 4a^4b^6$	When raising exponents to a power, keep the base and multiply the exponents.
4	$(xy)^a = x^a y^a$ $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$	$(a^4b)^3$ $= a^{12}b^3$ $\left(\frac{a^3}{b}\right)^3 = \frac{a^9}{b^3}$	When more than one base is raised to an exponent, each base is raised to the exponent. When a fraction is raised to an exponent, the numerator and denominator must be raised to that exponent.

RESOURCE 2

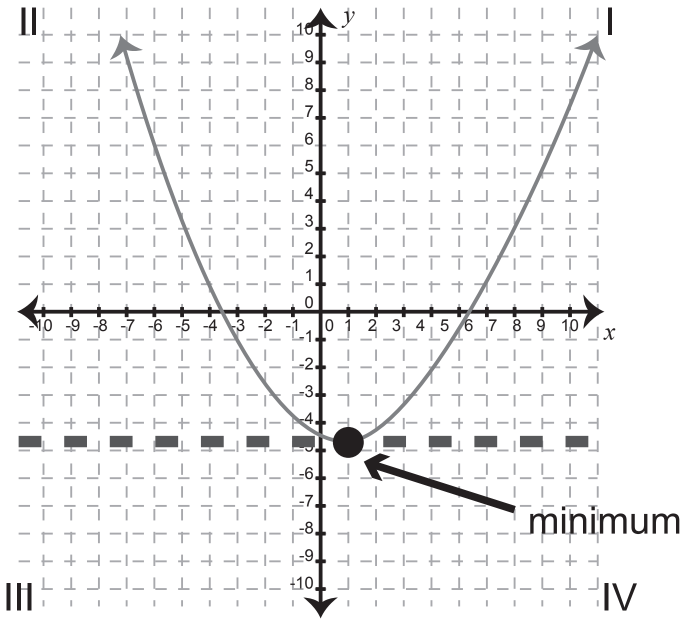
TOPIC 1: LESSON 1

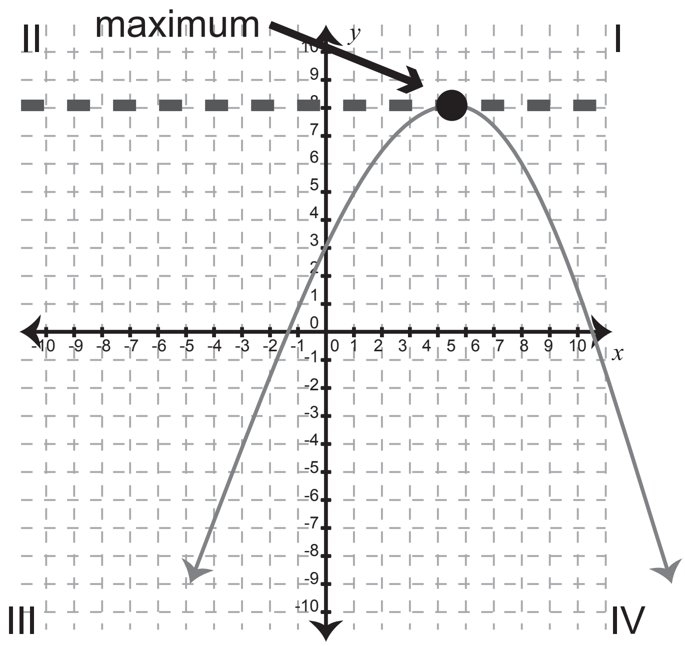
$x^0 = 1$	$(x^4 + 4)^0 + 3^0$ $= 1 + 1$ $= 2$	Any base raised to the power of zero is equal to 1. ($x \neq 0$ *as* 0º is undefined).
$x^{-a} = \frac{1}{x^a}$	$3x^{-2} = \frac{3}{x^2}$ and $\frac{3}{x^{-2}} = 3x^2$	A base raised to a negative exponent is equal to its reciprocal raised to the same positive exponent.

EQUATIONS AND INEQUALITIES

RESOURCE 3

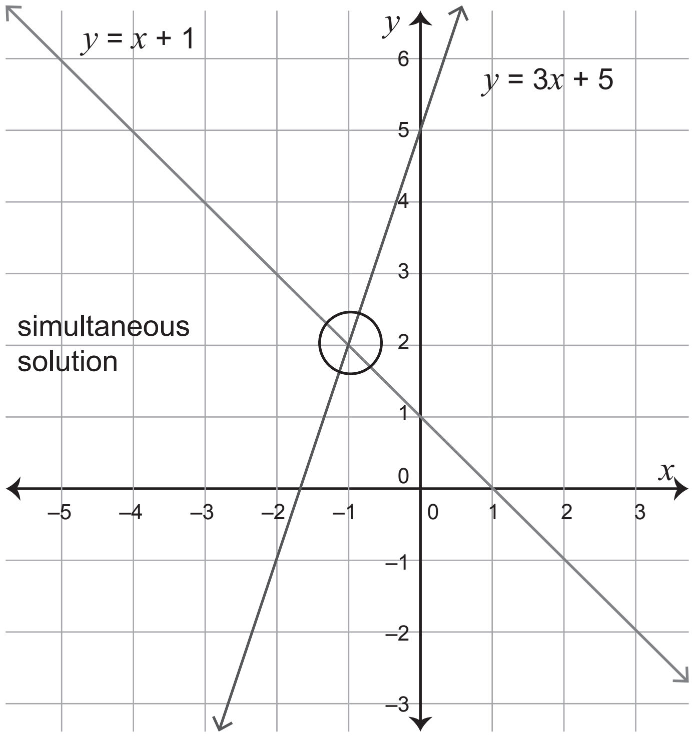
TOPIC 2: LESSON 1

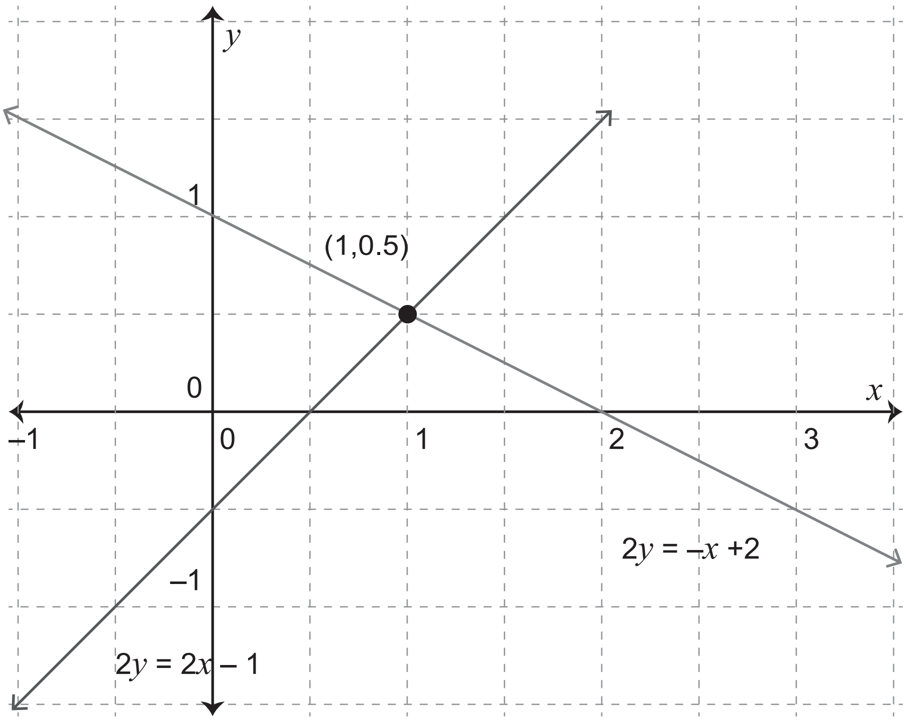




RESOURCE 4

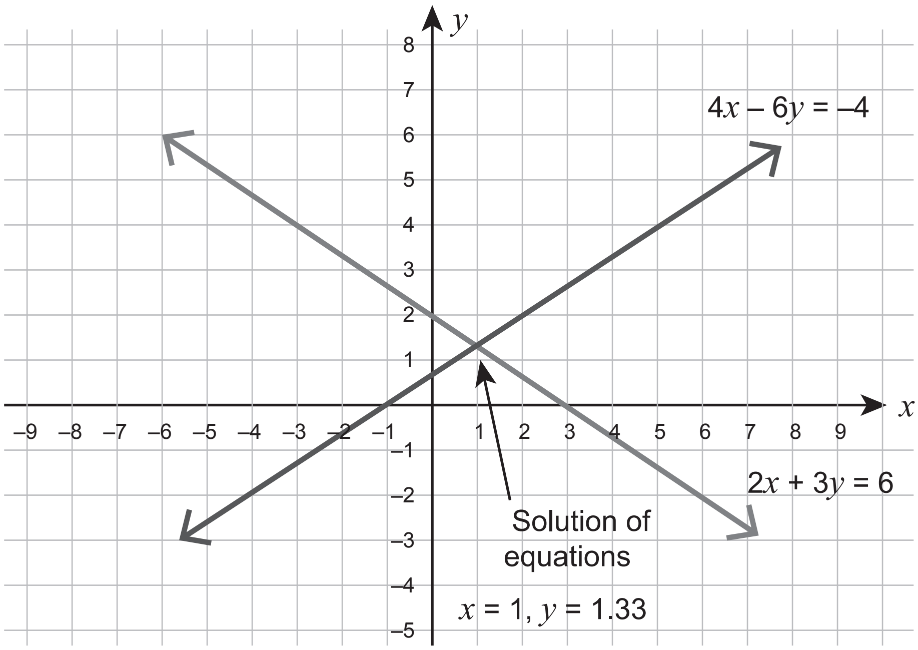
TOPIC 2: LESSON 6

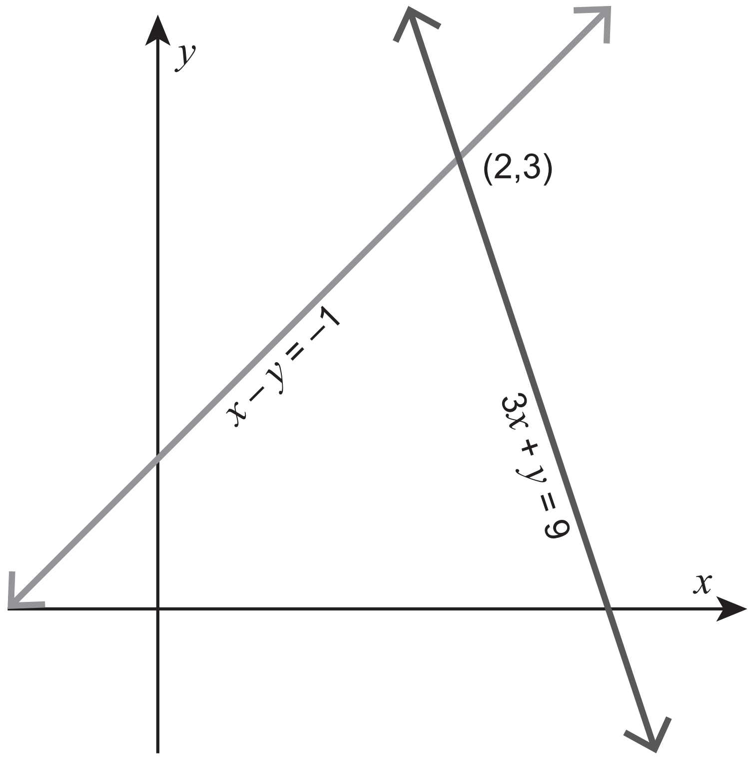




RESOURCE 5

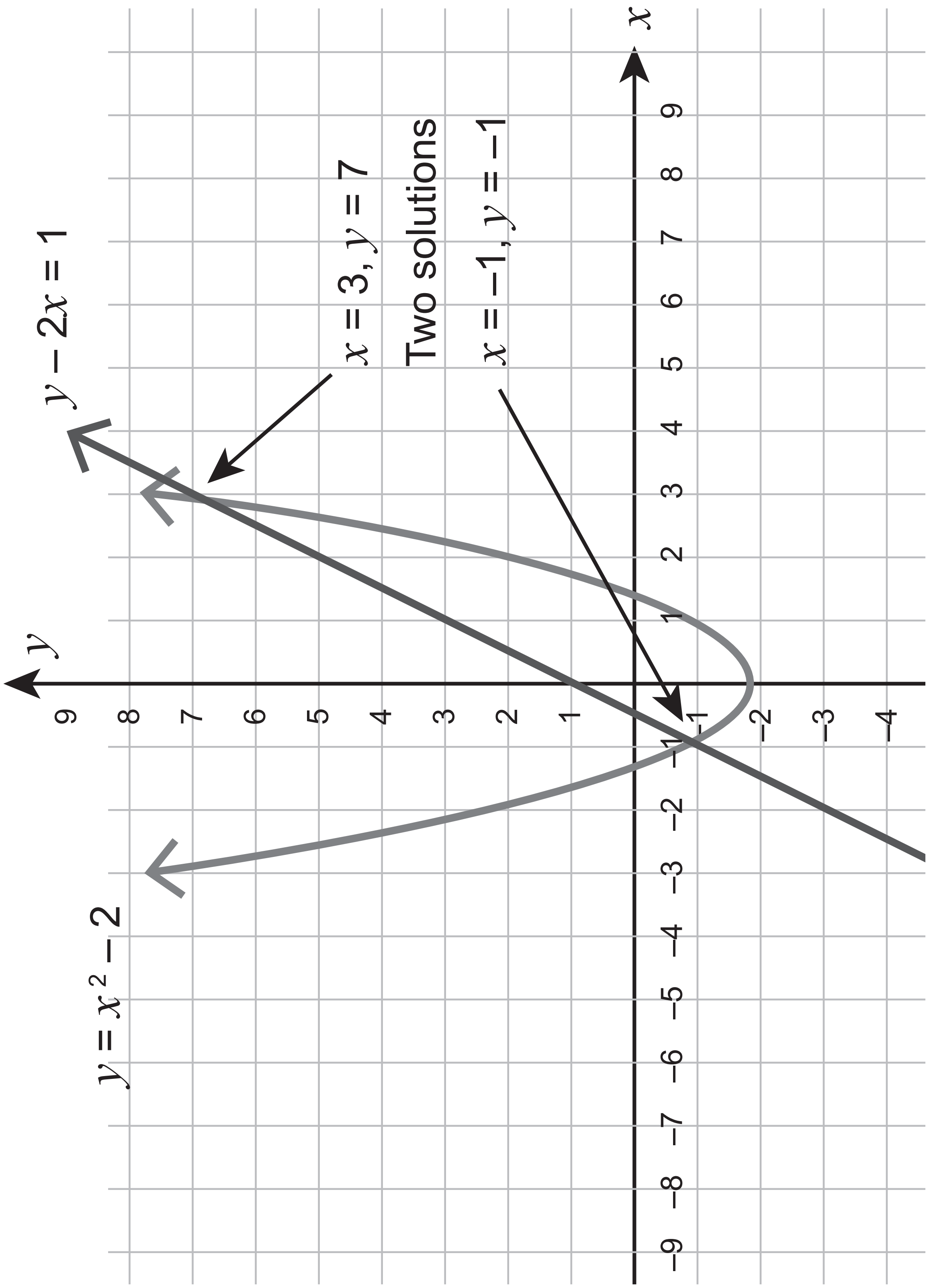
TOPIC 2: LESSON 6





RESOURCE 6

TOPIC 2: LESSON 6



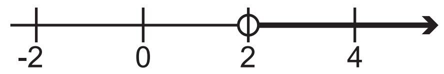
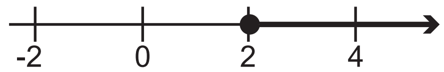
RESOURCE 7

TOPIC 2: LESSON 8

Inequalities, Interval Notation and Representation on a number line

Inequality sign	words	Open/closed dot	
>	Greater than	Open	
≥	Greater than or equal to	Closed	
<	Less than	Open	
≤	Less than or equal to	Closed	

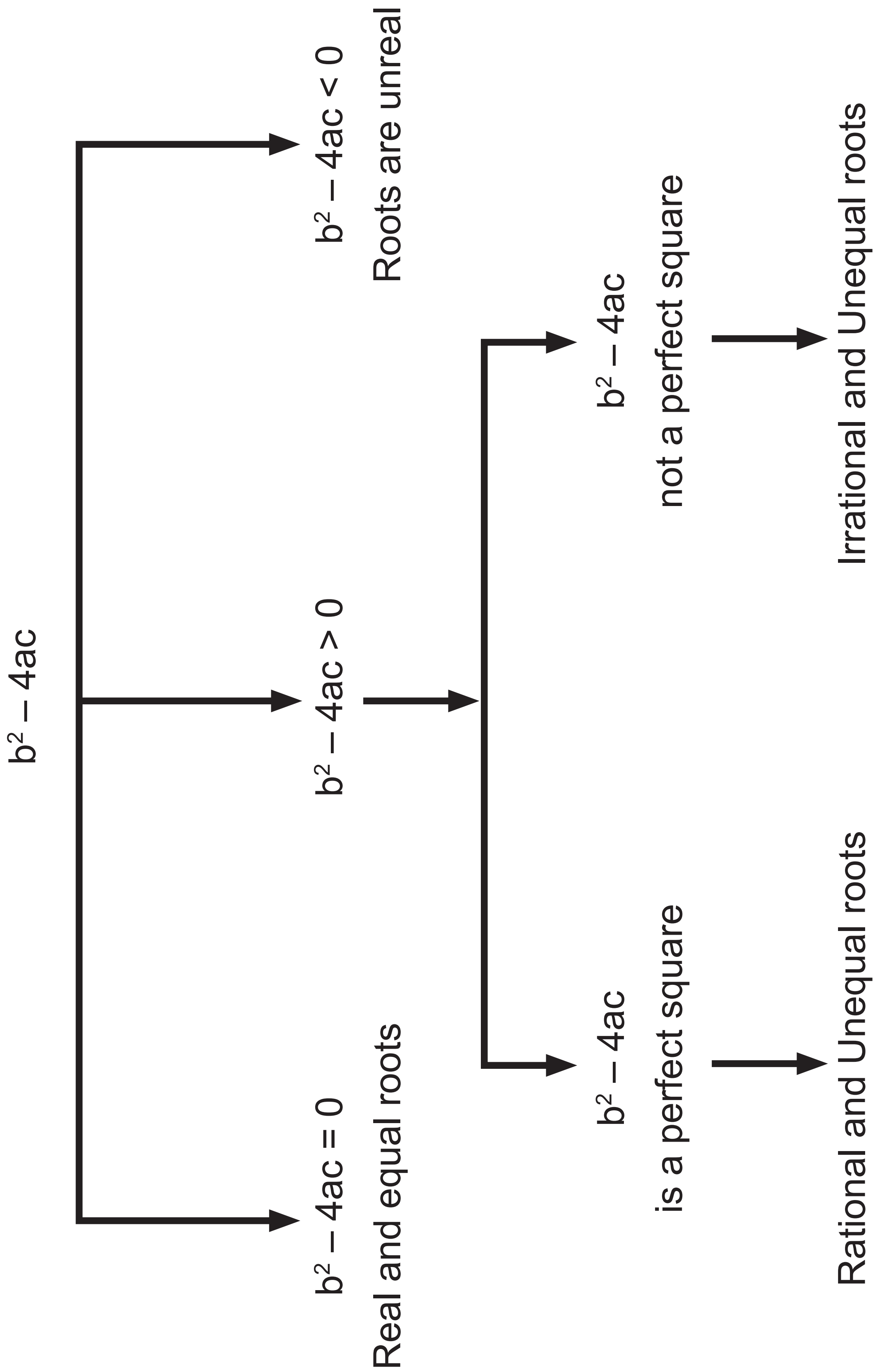
Examples:

Inequality	Interval notation	
$x > 2$	$x \in (2 ; \infty)$	
$x \geq 2$	$x \in [2 ; \infty)$	
$2 \leq x \leq 6$	$x \in [2 ; 6]$	
$2 < x < 6$	$x \in (2 ; 6)$	
$2 \leq x < 6$	$x \in [2 ; 6)$	
$2 < x \leq 6$	$x \in (2 ; 6]$	

Interval Notation is used to represent a set of Real Numbers as it is impossible to list them.

RESOURCE 8

TOPIC 2: LESSON 9

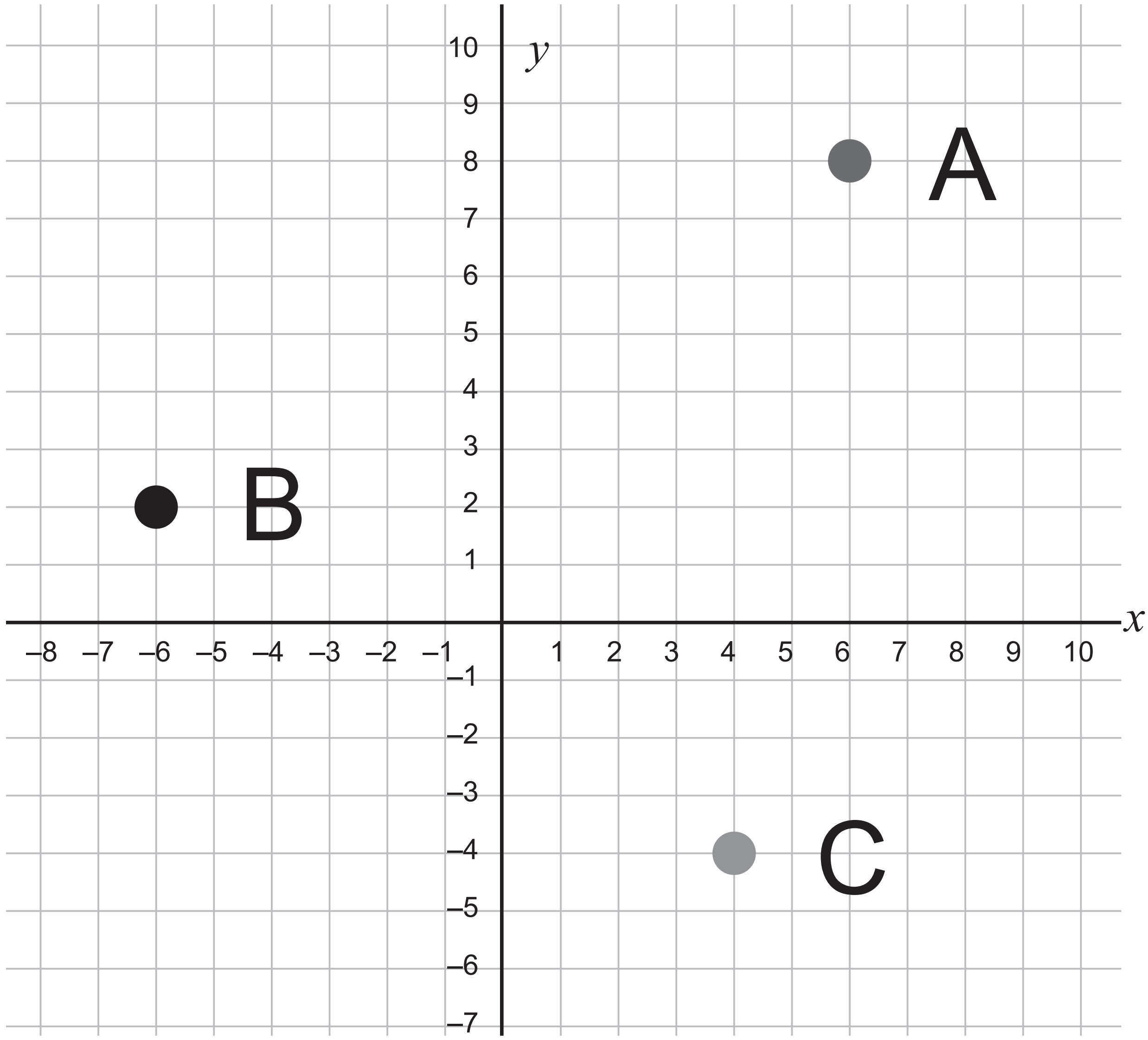


ANALYTICAL GEOMETRY

RESOURCE 9

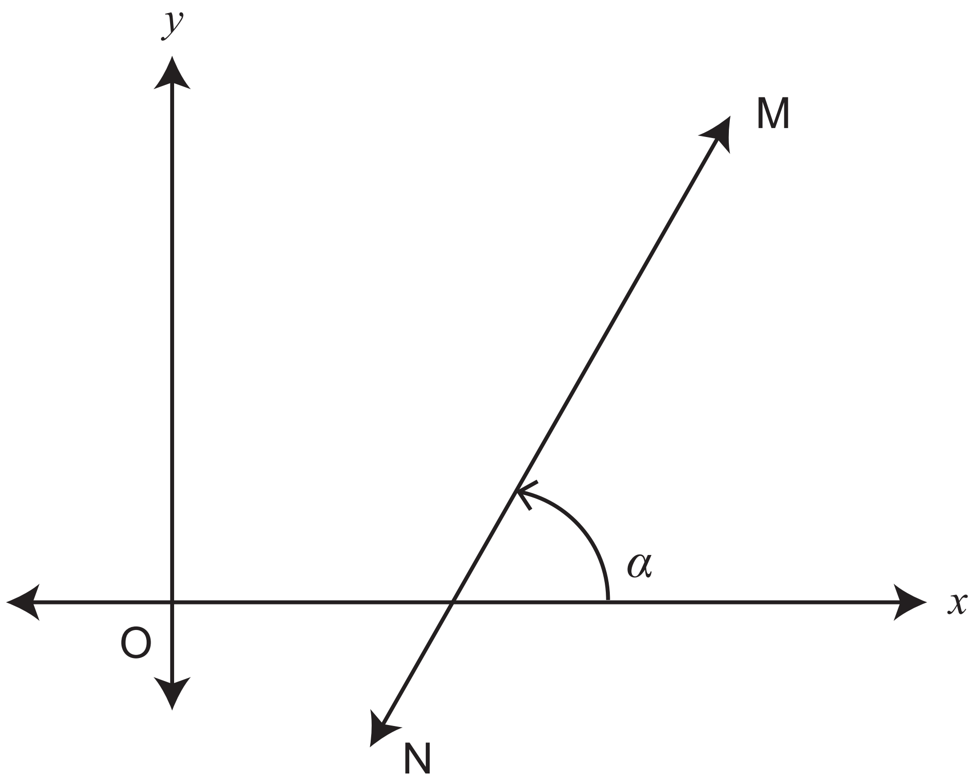
TOPIC 4: LESSON 2

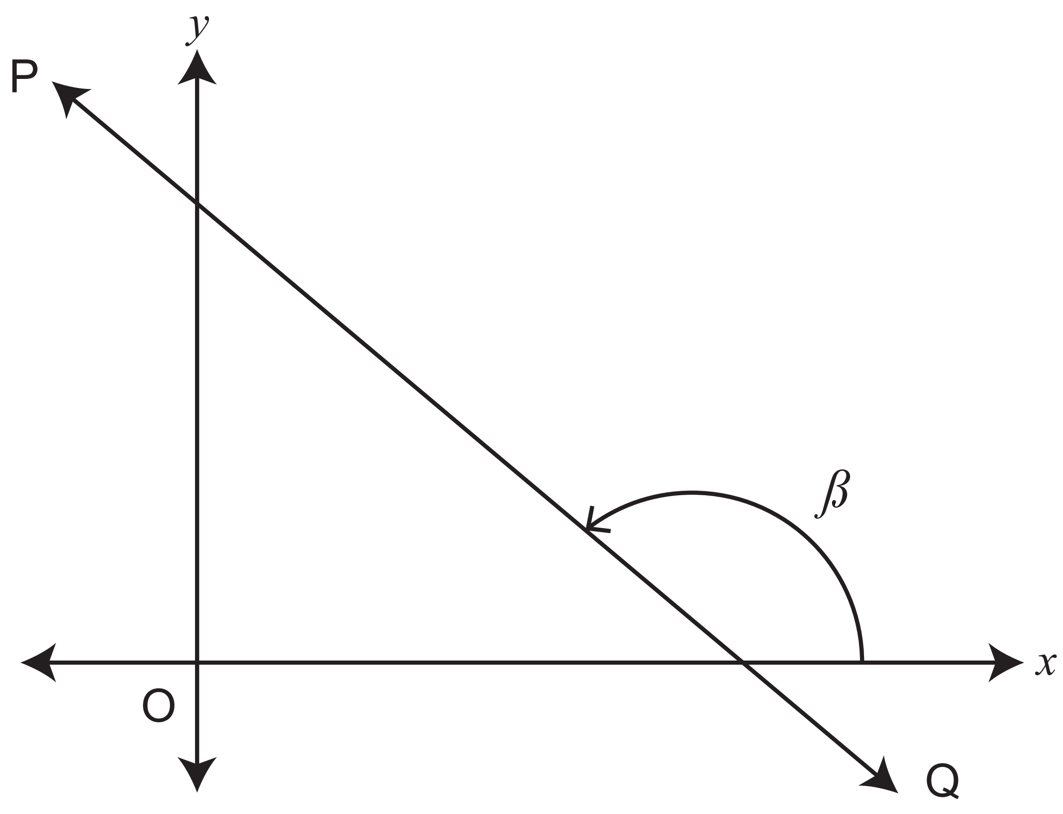
A (6 ; 8) ; B (–6 ; 2) ; C (4 ; –4)



RESOURCE 10

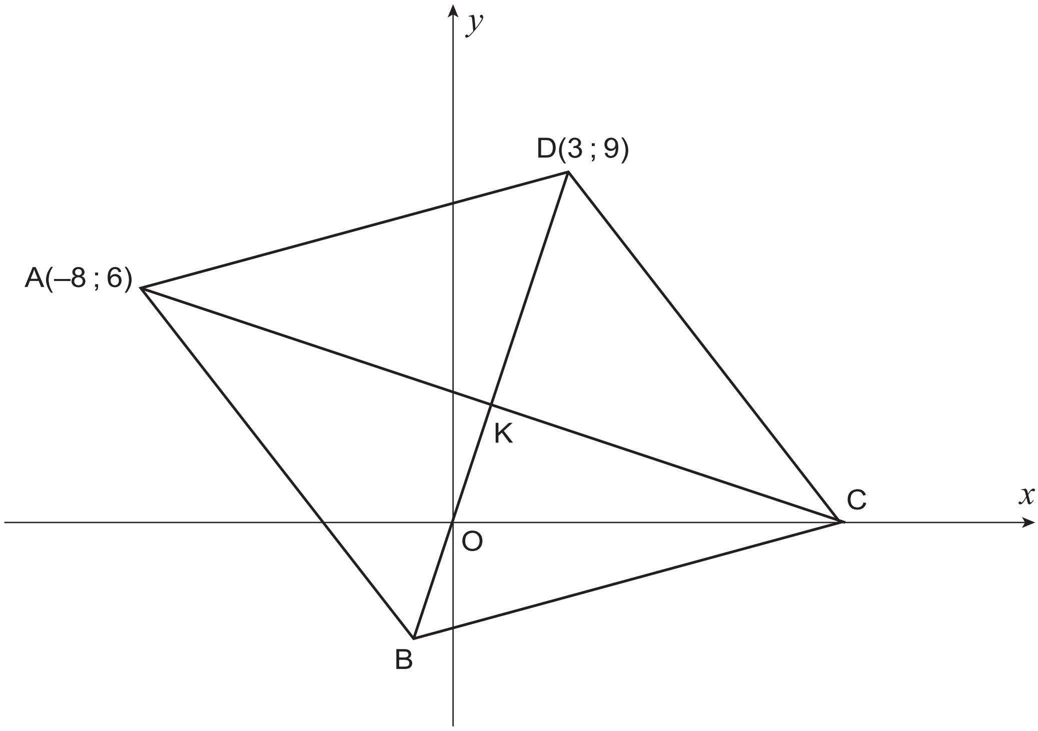
TOPIC 4: LESSON 4

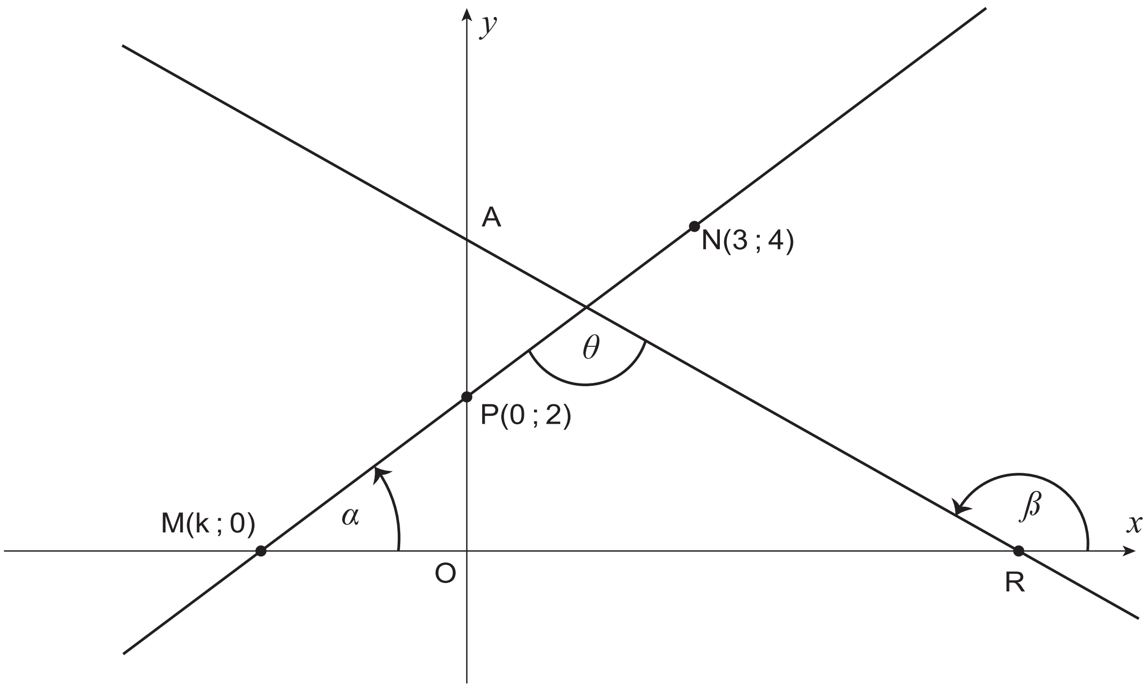




RESOURCE 11

TOPIC 4: LESSON 5



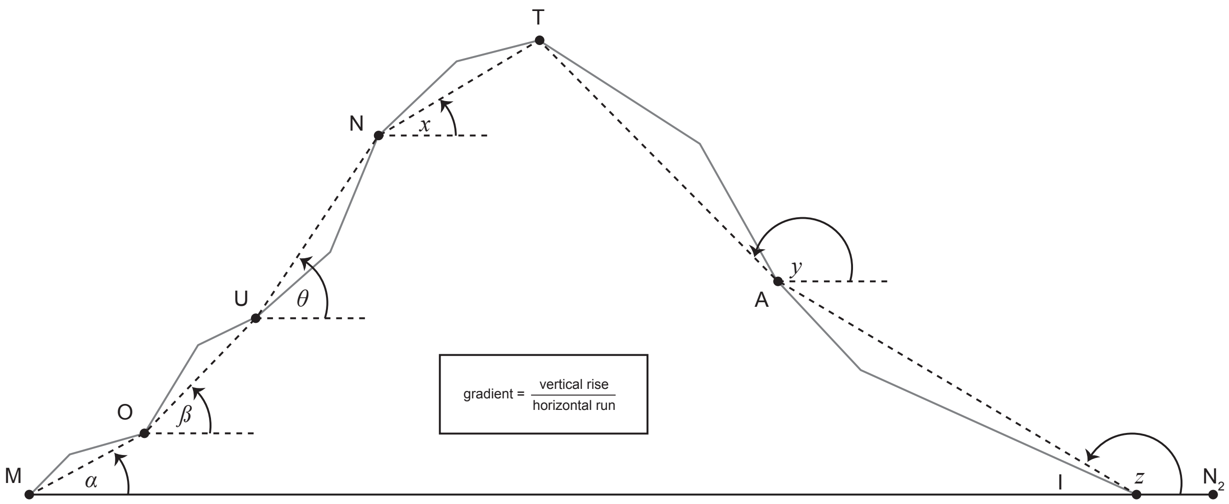


RESOURCE 12

TOPIC 4: LESSON 3

INVESTIGATION – ANALYTICAL GEOMETRY

NAME: ______________________________



Look carefully at the drawing of the mountain represented above. Imagine walking up the mountain from M and down the other side to N_2.

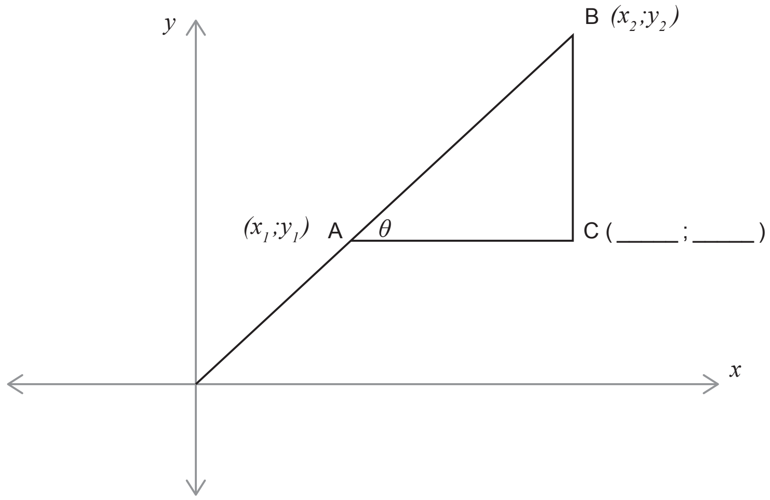
The dashed lines represent an average gradient between two points. Note that on the way up the mountain (M,O,U,N,T) all the gradients are positive and on the way down the mountain (A,I) the gradients are negative. The flat path at the end, to N_2, has a gradient of zero.

Note the angles made ($\alpha,\beta,\theta,x,y,z$). The angles are formed from a horizontal line (as angles are drawn if you use a protractor), in a clockwise direction to the line made representing the gradient. These are called angles of inclination.

Underline the correct answers in the following statements and fill in the missing spaces:

1. The lines *MO*, *OU*, *UN* and *NT* all slope **up/down** and therefore have a **positive/ negative** gradient. The angles of inclination made by the lines *MO*, *OU*, *UN* and *NT* all lie between _____ ° and ______° and are therefore ___________ angles. (4)
2. The lines *TA* and *AI* all slope **up/down** and therefore have a **positive/negative** gradient. The angles of inclination made by the lines *TA* and *AI* all lie between _____ ° and ______° and are therefore ___________ angles. (4)

Study the diagram below, then complete the questions that follow using your knowledge of Grade 10 Analytical Geometry and Trigonometry,



3. Fill in the coordinate at C in terms of x and y. (points A and B will be useful) (2)

4. In ΔABC,

BC (distance) = y_2 - _____ AC (distance) = _____________ (3)

5. From trigonometry, we know that:

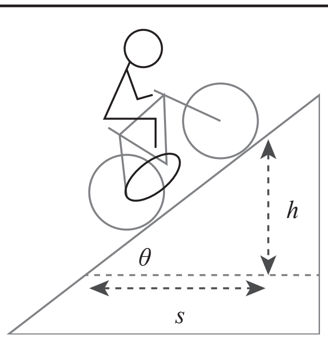
tan θ = ________

(use names of sides or $x/y/r$) (1)

∴ (using your answers from ABOVE (4), tan θ = _____ (2)

This is how we find the angle of inclination:

Using the gradient formula, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, which is essentially made up of $\dfrac{adj}{hyp}$ assists us in finding the angle of inclination of a straight line.

	tan θ = gradient $\left(\frac{h}{s}\right)$ tan θ = m (where θ is the angle of inclination and m is the gradient of the line)

Use the diagram of the mountain, and the formula from above. Answer the following questions:

6. If the first part of the path (*MO*) rises 3m over a horizontal distance of 27m, ind the angle of inclination (α). (3)

7. Find the angle of inclination (θ) of *UN* if it rises 35m over a horizontal distance of 2m. (3)

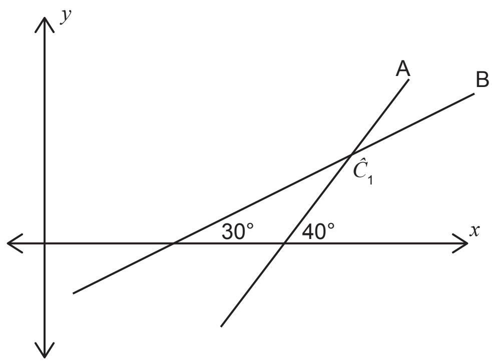
You will learn how to find an angle of inclination which is linked to a line with a negative gradient in class in a later lesson.

8. Underline the correct answers in each statement:

   a. If the gradient of a line is 0 (zero), the line will be **horizontal/vertical**. The inclination of this line is **0º**, **90º**, **180º**. (2)

   b. If the gradient of a line is undefined (perpendicular), the line will be **horizontal/vertical**. The inclination of this line is **0º**, **90º**, **180º**. (2)

9. Use the diagram below to answer the questions that follow:



Line A has an inclination of 40°. Line B has an inclination of 30°.

   a. Find the gradient of line A and line B. Round your answer to TWO decimal places.
   $m_A =$ $m_B =$ (2)

   b. i. Find $\hat{C}_1$ (1)

      ii. What theorem(s) did you use? (1)

Total: 30 marks

RESOURCE 13

Test Term 1

QUESTION	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Algebra	25	
2	Number Patterns	8	
3	Analytical Geometry	17	
	TOTAL	50	

QUESTION 1 **25 MARKS**

1.1 Simplify the following (no calculators may be used. All steps must be shown):

1.1.1 $32^{\frac{4}{5}}$ (3)

1.1.2 $\sqrt{125}+\sqrt{100}-\sqrt{75}$ (3)

1.2 Solve for x in the following:

1.2.1 $\sqrt{x+8}-10=0$ (2)

1.2.2 $3x^2 - 26x = 9$ by using the quadratic formula. (5)

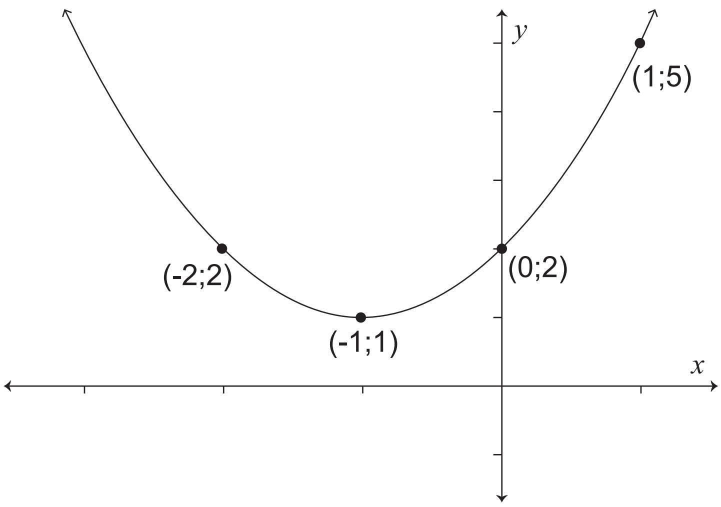
1.2.3 $3x^2 - 26x = 9$ by completing the square. (5)

1.2.4 $x^2 < 5x$ (4)

1.3 For which values of k will $2x^2 - 4x + k = 0$ have real roots. (3)

QUESTION 2 **8 MARKS**

2.1 Consider the following parabola with its given points:



2.1.1 Moving from left to right, write down the y-coordinates as a sequence. (1)

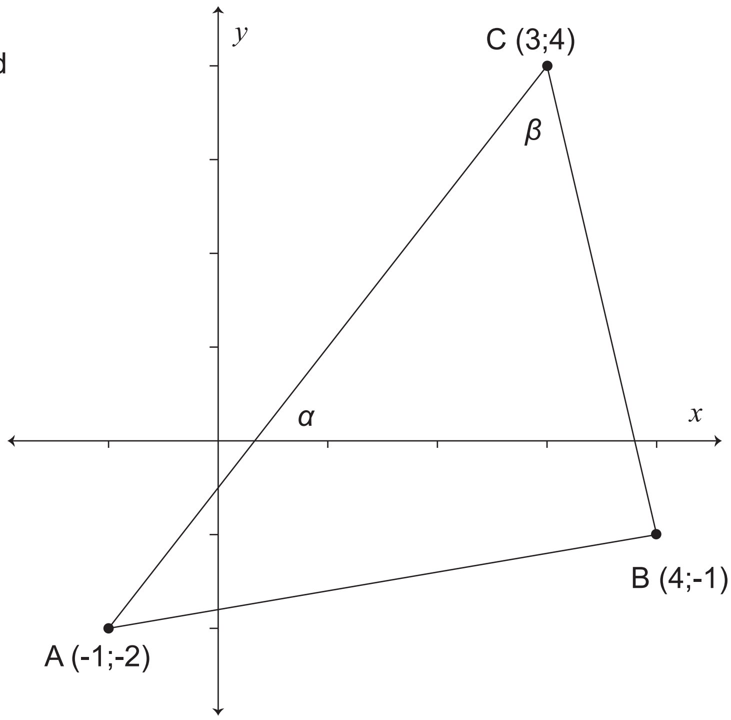
2.1.2 This sequence is quadratic since it has a common 2nd difference.
What is the value of this 2nd difference. (1)

2.1.3 Determine the nth term of the sequence. (4)

2.1.4 Determine the value of the 20th term. (2)

QUESTION 3 **17 MARKS**

3.1 Consider the triangle drawn to the right with vertices A, B, and C. α is the angle of inclination for line AC and β represents the angle $A\hat{C}B$



3.1.1 Determine the coordinates of the midpoint of line AC. (2)

3.1.2 Prove that AB is perpendicular to BC. (3)

3.1.3 Determine the equation of a line parallel to AB, passing through point C. (4)

3.1.4 Determine the value of α. (3)

3.1.5 Determine the value of β. (4)

3.1.6 Hence, determine the value of $C\hat{A}B$. (1)

RESOURCE 16

Memorandum Test Term 1

QUESTION	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Algebra	25	
2	Number Patterns	8	
3	Analytical Geometry	17	
	TOTAL	50	

QUESTION 1 **25 MARKS**

1.1 Simplify the following (no calculators may be used. All steps must be shown):

1.1.1 $32^{\frac{4}{5}}$ (3K)

$= \left(\sqrt[5]{32}\right)^4$ ✓

$= 2^4$ ✓

$= 16$ ✓

1.1.2 $\sqrt{125}+\sqrt{100}-\sqrt{75}$ (3R)

$= 5\sqrt{5} + 10 - 3\sqrt{5}$ ✓ ✓

$= 10 + 2\sqrt{5}$ ✓

1.2 Solve for x in the following:

1.2.1 $\sqrt{x+8} - 10 = 0$ (2R)

$\sqrt{x+8} = 10$

$x + 8 = 100$ ✓

$x = 92$ ✓

1.2.2 $3x^2 - 26x = 9$ by using the quadratic formula. (5R)

$x = \dfrac{26 \pm \sqrt{(-26)^2 - 4(3)(-9)}}{2(3)}$ ✓

$x = \dfrac{26 \pm \sqrt{784}}{6}$ ✓

$x = \dfrac{26 \pm 28}{6}$ ✓

$x = 9$ or $x = -\dfrac{1}{3}$ ✓ ✓

1.2.3 $3x^2 - 26x = 9$ by completing the square. (5C)

$x^2 - \dfrac{26}{3}x = 3$ ✓

$x^2 - \dfrac{26}{3}x + \dfrac{169}{9} = 3 + \dfrac{169}{9}$ ✓

$\left(x - \dfrac{13}{3}\right)^2 = \dfrac{196}{9}$ ✓

$x - \dfrac{13}{3} = \pm\dfrac{14}{3}$ ✓

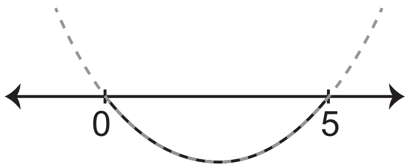
x = 9 or x = $-\dfrac{1}{3}$ ✓

1.2.4 $x^2 < 5x$ (4C)

$x^2 - 5x < 0$ ✓

$x(x - 5) < 0$ ✓ cv's: 0 ; 5

$0 < x < 5$ ✓ ✓



1.3 For which values of k will $2x^2 - 4x + k = 0$ have real roots. (3P)

$b^2 - 4ac > 0$ ✓

$(4)^2 - 4(2)(k) > 0$

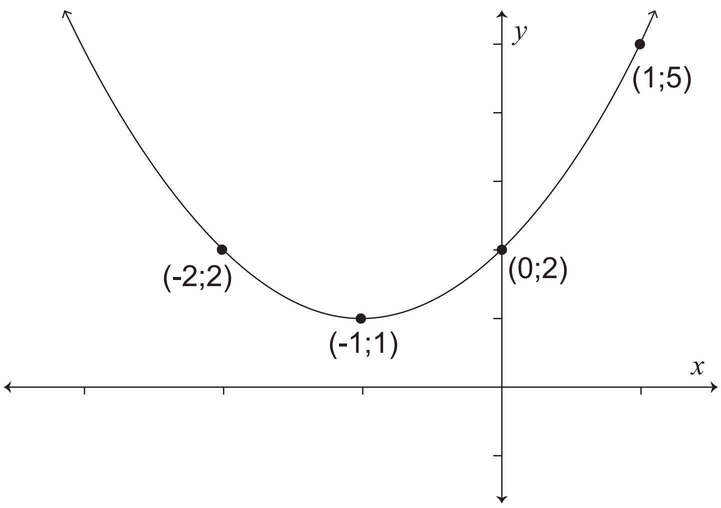
$16 - 8k > 0$ ✓

$-8k > -16$

$k < 2$ ✓

QUESTION 2 **8 MARKS**

2.1 Consider the following parabola with its given points:



2.1.1 Moving from left to right, write down the y-coordinates as a sequence. (1K)

2; 1; 2; 5... ✓

2.1.2 This sequence is quadratic since it has a common 2nd difference.
What is the value of this 2nd difference. (1K)

2nd difference = 2 ✓

2.1.3 Determine the nth term of the sequence. (4R)

$2a = 2$ $3a + b = -1$ $a + b + c = 2$

$a = 1$ ✓ $b = -4$ ✓ $c = 5$ ✓

$T_n = n^2 - 4n + 5$ ✓

2.1.4 Determine the value of the 20th term. (2R)

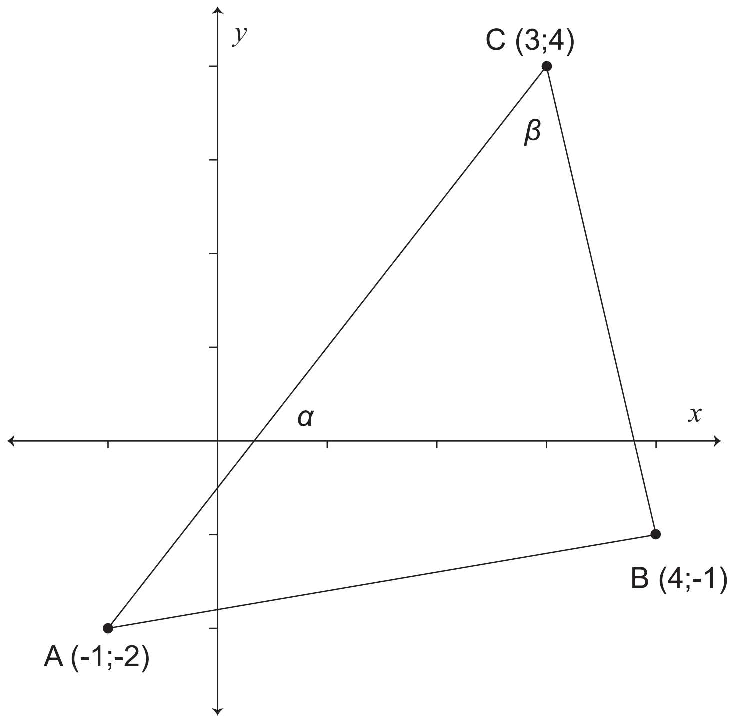
$T_n = n^2 - 4n + 5$

$T_{20} = (20)^2 - 4(20) + 5$ ✓

$T_{20} = 325$ ✓

QUESTION 3 17 MARKS

3.1 Consider the triangle drawn to the right with vertices A, B, and C. α is the angle of inclination for line AC and β represents the angle $A\hat{C}B$



3.1.1 Determine the coordinates of the midpoint of line AC. (2K)

$$M_{AB} = \left(\frac{-1+3}{2};\frac{-2+4}{2}\right) ✓$$

$$= \left(\frac{2}{2};\frac{2}{2}\right)$$

$$= (1;1) ✓$$

3.1.2 Prove that AB is perpendicular to BC. (3R)

$m_{AB} = \frac{-1+2}{4+1} = \frac{1}{5}$ ✓

$m_{BC} = \frac{4+1}{3-4} = -5$ ✓

$m_{AB} \times m_{BC} = \frac{1}{5} \times -5 = -1$ ✓

Therefore AB is perpendicular to BC

3.1.3 Determine the equation of a line parallel to AB, passing through point C. (4C)

$m = \frac{1}{5}$ (3;4)

$y - y_1 = m(x - x_1)$ ✓

$y - 4 = \frac{1}{5}(x - 3)$ ✓

$y = \frac{1}{5}x - \frac{3}{5} + 4$

$y = \frac{1}{5}x - \frac{9}{5}$ ✓ ✓

3.1.4 Determine the value of *α*. (3C)

$\tan \alpha = m_{AC}$

$m_{AC} = \frac{4+2}{3+1} = \frac{6}{4} = \frac{3}{2}$ ✓

$\tan \alpha = \frac{3}{2}$ ✓

$\alpha = 56{,}31°$ ✓ ✓

3.1.5 Determine the value of *β*. (4P)

$\tan \theta = m_{BC}$

$\tan \theta = -5$ ✓

$\theta = -78.69° + 180°$

$\theta = 101{,}3°$ ✓

$\beta = 101{,}3° - 56{,}31°$ (ext < of ∆) ✓

$\beta = 45°$ ✓

3.1.6 Hence, determine the value of $C\hat{A}B$. (1K)

$C\hat{A}B = 180° - 90° - 45°$

$C\hat{A}B = 45°$ ✓